

The thermal Casimir effect for rough metallic plates

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We propose a new theory of thermal Casimir effect, holding for the experimentally important case of metallic surfaces with a roughness having a spatial scale smaller than the skin depth. The theory is based on a simple phenomenological model for a rough conductor, that explicitly takes account of the fact that ohmic conduction in the immediate vicinity of the surface of a conductor is much impeded by surface roughness, if the amplitude of roughness is smaller than the skin depth. As a result of the new model, we find that surface roughness strongly influences the magnitude of the thermal correction to the Casimir force, independently of the plates separation. Our model, while consistent with recent accurate measurements of the Casimir force in the submicron range, leads to a new prediction for the not yet observed thermal correction to the Casimir force at large plates separation. Besides the thermal Casimir problem, our model is relevant for the correct theoretical interpretation of current experiments probing other proximity effects between conductors, like radiative heat transfer and quantum friction.

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The Casimir effect [1] provides physicists with the rare opportunity to investigate in a laboratory the physics of the quantum vacuum [2], a subject that has recently attracted much interest for a number of reasons. One one hand, the possibility exists that the recently discovered accelerated expansion of Universe might be explained by a cosmological term originating from quantum vacuum effects [3]. Another reason of interest comes from current experiments on non-newtonian forces at the sub-micron scale [4]. As its is well known, Casimir forces are the dominant forces at this scale, and therefore determining them very accurately is indispensable in order to obtain experimental bounds on possible non-Newtonian forces [5]. Finally, we would like to mention that the Casimir effect, and more in general van der Waals forces [6], are now finding new applications in nanotechnology [7].

In the field of Casimir physics, an important and much controversial problem is that of determining how the Casimir force between two metallic bodies is modified by the temperature of the environment [1]. Apart from its intrinsic interest as a problem in theory of quantum electromagnetic (e.m.) fluctuations, addressing this problem is important because many experiments on non-newtonian forces at the submicron scale use metallic surfaces at room temperature [5]. We remark that, while the Casimir pressure has now been measured accurately [5, 8], the thermal contribution to the Casimir effect has not yet been detected as we write, and indeed there are at present several ongoing and planned experiments, to measure it [9]. Surprisingly, the theory of the thermal Casimir effect for metallic bodies turned out to be rather complicated, and at present there exist several conflicting approaches that give widely different predictions for the magnitude of the effect [1].

In this Letter we present a new theory of the thermal Casimir effect for conductors. The distinctive new feature of the theory consists in the central role played in our approach by the surface roughness of the conductor. The presence of roughness is indeed a general feature of current Casimir experiments, which use metallic surfaces presenting surface roughness with a characteristic amplitude h of a few nanometers [1, 5]. Until now, it has always been thought that for separations a between the surfaces much larger than the roughness amplitude h , surface roughness implies only a small correction of order $(h/a)^2$ [1]. We argue that this conclusion, while correct for insulators, is not warranted for metallic surfaces at finite temperature, if the roughness h is smaller than the skin depth δ , because then account must be taken of the fact that surface irregularities impede ohmic conduction in the immediate vicinity of the surface, a fact noted already by Pippard [10] long ago. Using a simple phenomenological model to account for this effect, we then find that *independently of the separation a* , the magnitude of the thermal correction to the Casimir force strongly depends on the surface roughness. While consistent with a recent precise measurement [5] of the Casimir pressure in the submicron range, our theory leads to a new prediction for the magnitude of the not yet measured thermal correction to the Casimir force at large separations. We note that the application range of our model extends to other proximity effects originating from e.m. fluctuations, like radiative heat transfer [11] and quantum friction between closely space metallic surfaces [12], that are currently under intense investigation, but we shall not touch upon these issue in this Letter.

All current approaches to the thermal Casimir effect are based on the theory of the Casimir effect developed

long ago by Lifshitz [13], on the basis of Rytov's general theory of e.m. fluctuations [14]. Lifshitz-Rytov theory works very well in the case of media that can be characterized by electric and magnetic permittivity tensors, $\epsilon_{ij}(\omega)$ and $\mu_{ij}(\omega)$ that depend only on the frequency, but are independent of the wave vector \mathbf{k} , i.e. for media that exhibit only time-dispersion but not space-dispersion. According to this theory, the Casimir pressure P between two identical plane-parallel (possibly stratified) slabs at temperature T , separated by an empty gap of width a , is:

$$P = -\frac{k_B T}{2\pi^2} \sum_{l \geq 0}' \int_0^\infty d^2 \mathbf{k}_\perp q_l \sum_{\alpha=\text{TE, TM}} \left(\frac{e^{2a q_l}}{r_\alpha^2(i\xi_l, \mathbf{k}_\perp)} - 1 \right)^{-1}, \quad (1)$$

where the prime over the l -sum means that the $l = 0$ term has to be taken with a weight one half, \mathbf{k}_\perp denotes the projection of the wave-vector onto the plane of the plates and $q_l = \sqrt{k_\perp^2 + \xi_l^2}$, where $\xi_l = 2\pi k_B T l / \hbar$. The quantities $r_\alpha(i\xi_l, \mathbf{k}_\perp)$ denote the reflection coefficients of the slabs for α -polarization (for simplicity, we do not consider the possibility of a non-diagonal reflection matrix), evaluated at complex frequencies $i\xi_l$. For the case of single-layer homogeneous slabs, with permittivity $\epsilon(\omega)$, r_α are the familiar Fresnel reflection coefficients. The above formula for the Casimir pressure is regularly used also in the case of metals, that can be characterized by a (complex) conductivity $\sigma(\omega)$, in which case one has for the permittivity the expression $\epsilon(\omega) = 1 + 4\pi i \sigma(\omega)/\omega$. The difficulties found recently in the study of the thermal Casimir effect for metals originate from the fact that in order to evaluate the $l = 0$ term of Eq. (1) one needs extrapolate the reflection coefficients r_α to zero frequency. Different prescriptions for doing this have been proposed, leading to largely different predictions for the thermal correction to the pressure, depending on the resulting magnitude of the $l = 0$ term. For example, the approach of Ref.[15] adopts the Drude model

$$\epsilon_D(\omega) = 1 - \Omega_P^2 / [\omega(\omega + i\gamma)], \quad (2)$$

where Ω_P is the plasma frequency and γ the relaxation frequency accounting for ohmic conductivity. When this model is used, one finds that the $l = 0$ term for TE polarization gives zero contribution, and the predicted thermal correction is much larger than what one obtains for the ideal case of perfect reflectors (corresponding to taking $r_\alpha^2 = 1$ in Eq. (1)). Recently, it has been shown that this model is inconsistent with the experiment [5] and moreover it has been noted that, in the case of perfect lattices, the Drude model leads to thermodynamical inconsistencies at low temperature [16]. On the contrary, the approach based on the plasma model

$$\epsilon_P(\omega) = 1 - \Omega_P^2 / \omega^2, \quad (3)$$

(augmented by a six-oscillator contribution accounting for interband transitions), has been shown to be consis-

tent with the experiment [5], and it is immune of thermodynamic inconsistencies.

All the above is for perfectly plane-parallel plates while, as we said earlier, real experiments involve rough surfaces, with amplitude h typically in the range of a few nanometers. In the existing literature, the effect of the slabs roughness has always been regarded as a *geometric* problem, involving as the only relevant parameter the geometric quantity h/a . For $h \ll a$, the problem has been studied by perturbative means, for example the so-called Proximity Force Approximation (PFA) which corresponds to taking a suitable average of the Lifshitz formula result, across the surface of the plates [1]. The result of the PFA is that for $h \ll a$, roughness gives only a small fractional correction of order $(h/a)^2$ to the Casimir force between two perfectly flat plates (see for example Ref. [5]). Below we show that such a conclusion is not correct however, if the roughness profile varies rapidly along the surface of the conductor, over distances equal to the skin depth δ of the e.m. fields, as it is usually the case in current Casimir experiments.

Our analysis of the effect of roughness starts from the remarks made a long time ago by Pippard in his investigations of the skin effect [10]. Pippard noted that at frequencies in the GHz region, the presence of microscopic scratches on the surface of a metal can determine a significant increase of the surface resistance, by forcing the electric currents to flow along longer, distorted paths inside the metal. We considered that these remarks acquire even greater importance if the irregularities of the surface are smaller than the skin depth δ . If such a conductor is placed in a tangential alternating electric field, the induced ohmic currents in the immediate vicinity of the surface will be unable to follow the rapid distortions of the surface, with the result that barely any current will be found in the outer layer of thickness h of the conductor. This is like water flowing inside a large pipe whose inner surface presents small bumps. The water contained in these bumps does not take part in the flow, and remains still. We were thus led to conceive that *the surface layer of thickness h of a rough conductor does not behave like the deeper part of the conductor, and in fact, with regards to tangential electric fields, it rather resembles a dielectric, having a negligible real conductivity $\sigma(\omega)$* . A detailed investigation of the response function of a rough metallic surface, according to the above picture, is likely to be a rather complicated problem, because when h is comparable to or less than the electron mean free path l at room temperature (for gold l is around 20 nm), spatial dispersion is expected to be relevant. In this Letter we shall content ourselves with proposing a simple phenomenological model, suggested by the previous qualitative considerations. We neglect the possible effect of spatial dispersion near the metal's surface, and we model a (thick) rough metallic plate as a *plane-parallel two-layer system*, consisting of a thick slab with permittivity $\epsilon(\omega)$,

describing the inner part of the conductor, covered by a thin plane-parallel uniform layer of thickness h with a *different* permittivity $\epsilon_{\text{surf}}(\omega)$, modelling its rough surface. For the bulk permittivity $\epsilon(\omega)$ we take the usual Drude model in Eq. (2), while for the thin outer layer we take the plasma model:

$$\epsilon_{\text{surf}}(\omega) = 1 - \Omega_{\text{surf}}^2/\omega^2, \quad (4)$$

with

$$\Omega_{\text{surf}}^2 = f \Omega_{\text{p}}^2, \quad (5)$$

where f is the fraction of the plane-parallel layer of width h really occupied by the metal. We now explain these choices. Modelling the rough surface as a uniform plane-parallel layer is reasonable, because in typical Casimir experiments at room temperature the wavelengths $\lambda \simeq a$ and the skin depths δ of the e.m. fields are both much larger than h , and therefore the e.m. fields effectively see the rough surface as flat. The choice of the plasma-model for ϵ_{surf} appears as the simplest one, because the plasma model is what one obtains from the Drude model, after one sets to zero the real part of the conductivity σ , in such a way that the surface layer behaves as an insulator, as we argued it is the case. The chosen value of Ω_{surf}^2 in Eq. (5) depends on the fact that the average density n_{surf} of free electrons within the rough layer is smaller by a factor f than the bulk value n . Upon recalling that the square of the plasma frequency is related to the density n of free electrons by the well known relation $\Omega^2 = 4\pi n e^2/m$, where e and m denote the electron and (effective) mass, respectively, one arrives at Eq. (5). In principle, for a given rough surface, it should be possible to compute the values of h and f , starting from a detailed microscopic theory. In practice, we shall regard h and f as *phenomenological* parameters to be determined by comparison with experimental data.

The two layer structure for a rough metallic plate is what distinguishes our model from the models that have appeared in the recent literature, which all use a single dielectric function $\epsilon(\omega)$ for the entire plate, the various approaches differing in the choice of $\epsilon(\omega)$. It is clear that for sufficiently large h our model is expected to reproduce the results of the plasma model used in Ref. [5], while for $h \rightarrow 0$ it should resemble the Drude model of Ref.[15]. To see exactly how things go, we now compute the thermal Casimir force between two identical rough plates, using our model. This is done simply by substituting into Lifshitz formula Eq. (1) the appropriate reflection coefficients r_α for our two-layer model of a rough plate:

$$r_\alpha = \frac{r_\alpha^{(01)} + r_\alpha^{(12)} \exp(-2hs^{(1)})}{1 + r_\alpha^{(01)} r_\alpha^{(12)} \exp(-2hs^{(1)})}, \quad (6)$$

where $\epsilon^{(0)} = 1$, $\epsilon^{(1)} = \epsilon_{\text{surf}}(\omega)$, $\epsilon^{(2)} = \epsilon(\omega)$, $s^{(j)} = \sqrt{\epsilon^{(j)}(i\xi_l)\xi_l^2/c^2 + k_\perp^2}$, and $r_\alpha^{(ij)}$ are the usual Fresnel reflection coefficients for the interface ij . In what follows

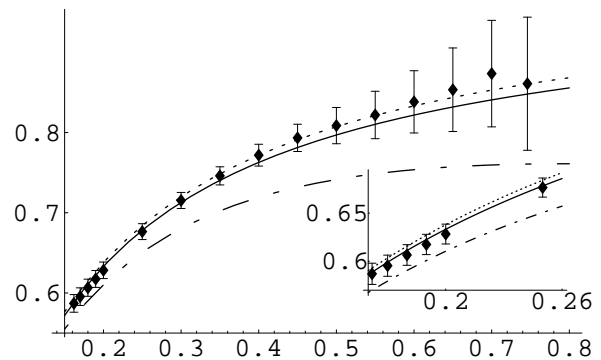


FIG. 1: Experimental data for the reduction factor η from Ref. [5] (diamonds). The inset shows a magnified view of the picture for small separations. The solid and dotted lines are fits using our theory, with $h = 11$ nm and $f = 0.9$, for $T = 300$ K (solid line) and $T = 0$ K (dotted line). The dot-dashed line is for the Drude model approach at $T = 300$ K. Distances d are in microns. See text for explanations

we shall plot the Casimir pressure as a function of the *average* separation d between the plates, that is related to a as $d = a + 2h(1 - f)$. Indeed d is a more meaningful quantity to consider, because it coincides with the plates separation that is usually measured in the experiments.

As a check of our theory, we tried to fit a recent (indirect) accurate measurement of the Casimir force between two plane-parallel gold surfaces, using a micromachined oscillator, reported in Ref. [5]. The experimental data are shown in Fig. 1, in terms of the reduction factor $\eta = P/P_{\text{id}}$, where $P_{\text{id}} = (\pi^2 \hbar c)/(240d^4)$ is the Casimir pressure for two ideal metallic plates, at zero temperature. In our computations, we took $\hbar\Omega_P = 8.9$ eV and $\hbar\gamma = .0357$ eV, which are the values used in [5]. As the most accurate measurements were performed at rather small separations, around 200 nm, interband transitions of core electrons give a sizable contribution to the permittivity, and we therefore added both to ϵ and ϵ_{surf} the six-oscillator expression $\epsilon_{\text{IB}}(\omega)$ accounting for interband transitions, that was used in Ref.[5]. Fig.1 shows two fits with our model for $h = 11$ nm and $f = 0.9$, for $T = 300$ K (solid-line) and $T = 0$ (dotted line). We see that both curves are consistent with all the measurements in the entire range of distances, ranging from 162 nm to 746 nm, with errors well inside the 95 confidence interval reported in Ref. [5]. The values for h and f obtained from the fit are nicely consistent with the degree of roughness of the Au surfaces reported in Ref. [5], which appears as encouraging in favor of our model. As we see, the precision of the measurements is not sufficient to discriminate the $T = 0$ and $T = 300$ K curves, and therefore this experiment does not detect the thermal contribution to the Casimir force. We remark that the data rule out the Drude-approach of Ref. [15] (dot-dashed line), while they are consistent with the plasma-model approach ad-

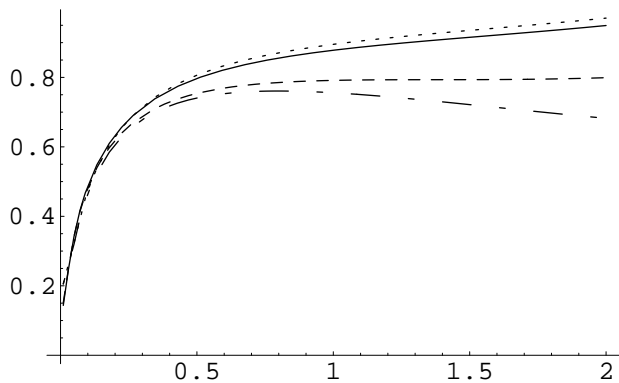


FIG. 2: Plots of the reduction factor η for Au plates at $T = 300$ K, as a function of the *average separation* d (in microns). The solid and dashed curves are for our theory, with $h = 11$ nm and $f = 0.9$ (solid curve), $h = 2$ nm and $f = 0.5$ (dashed curve). The dotted curve is for the plasma-model of Ref. [5] and the point-dashed curve for the Drude model of Ref. [15].

vocated in Ref. [5], and in fact for this experiment our model gives results that are very close to those obtained by means of the plasma model.

It is now interesting to consider how the magnitude of the thermal correction depends on the roughness at larger separations, and in Fig. 2 we show plots of the reduction factor for different degrees of roughness. The curves were computed for gold, with $\hbar\Omega_P = 8.9$ eV, and $\hbar\gamma = .0357$ eV, for $T = 300$ K. The dotted and dot-dashed line correspond, respectively, to the (generalized) plasma approach of Ref.[5] and to the Drude approach of Ref.[15]. The solid and dashed lines were computed using our model, with $l = 11$ nm, $f = 0.9$ (solid) and $l = 2$ nm, $f = .5$ (dashed). Fig. 1 lends itself to several important comments. First of all we see that, according to our model, surface roughness does influence strongly the Casimir pressure also at large separations, contrary to the PFA picture. Second, neither the plasma model, nor the Drude model are generally valid. The plasma model may be expected to be valid only for "very" rough surfaces, having a roughness amplitude h around ten nm, like those used in the experiment in Ref. [5]. On the contrary, the Drude model should be valid for very smooth surfaces, with a roughness amplitude h well below two nm. For intermediate values of the roughness amplitude, neither theory should be adequate and perhaps the simple model proposed in this Letter provides a better approximation.

In conclusion, we have presented a new phenomenological theory for the thermal Casimir effect between two rough metallic plates. According to this theory, surface roughness influence strongly the thermal correction to the Casimir force at all separations a . The theory is expected to be valid for surfaces with a roughness having a characteristic spatial scale h below the skin depth δ , which represents the standard experimental situation. For smoother surfaces, having slow profile variations on

the scale of the skin depth, the usual approximations based on the PFA are expected to be correct, and then roughness should only give a small correction of order $(h/a)^2$. We do not know yet if our two-layer model is free from the thermodynamical inconsistencies at low temperature that plague the Drude model [16], and we plan to investigate this important problem in a future work. Another important remark is that the model presented in this Letter can be used to obtain predictions for other proximity effects, originating from thermal e.m. fluctuations, like for example the power S radiative heat transfer between two metallic plates at different temperatures, separated by an empty gap. Investigating this problem would be very interesting, because that would provide us with another check of the correctness of our model for a rough metallic surface. This subject also will be addressed in future work.

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